HIGHER PRELIM REVISION

1. Find $\frac{dy}{dx}$ when (a) $y = \left(\frac{3x+4}{2x^2}\right)$ (b) $y = \left(x - \frac{1}{x}\right)^2$

2. Find the equation of the tangent to the curve $y = \frac{1}{2x}$ at x = 1

3. If $\cos\theta = \frac{5}{13}$ and $0 \le \theta \le 90°$ find the values of (a) $\sin 2\theta$ and (b) $\cos 2\theta$

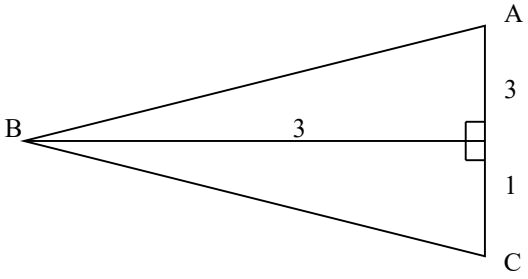
4. Show that the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 4x - 1$ is never decreasing.

5. For what values of c does the equation $c(x^2+1) = 2x$ have equal roots?

6. Solve the trig equation $5\cos 2x° + \cos x° + 2 = 0$ for $0 \le x \le 360°$

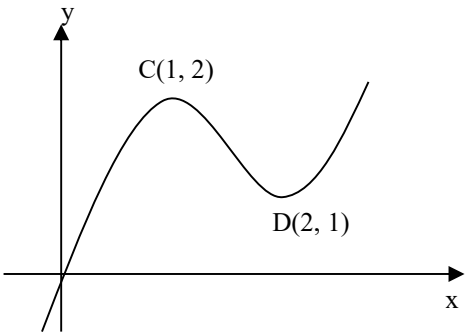
7. Using the diagram shown prove that

$\sin A\hat{B}C = \frac{2\sqrt{5}}{5}$



8. The sketch graph of a cubic function $y = f(x)$ is shown.

Sketch the graph of $f'(x)$.



9. A closed rectangular box is to be used with a special metal strip around all of its edges. The box has square ends and a volume of 216cm^3. Show that if each square end has length of side x cm, the total length L of the strip required is given by

$$L = \frac{8x^3 + 864}{x^2}$$

Find x so that L is a minimum.

10. If $k = \frac{x^2+2}{2x+1}$ rewrite this as a quadratic in x and hence find k so that this equation has equal roots.

11. Find the equation of the line joining the points (0, -1) and (3, 5). Find the points where the line cuts the circle $x^2 + y^2 = 2$

12. A circle, centre C, has equation $x^2 + y^2 - 4x + 6y - 12 = 0$.

(a) Find the equation of the tangent at the point A (5, 1) on this circle.
(b) Show that the line through the point P (1, 4) at right angles to this tangent has equation $3y - 4x = 8$ and show that this line is also a tangent to the circle.

13. If $f(x) = x + 4$ and $g(x) = x^2 - x - 3$

(a) Find $f(g(x))$ and $g(f(x))$
(b) Solve the equation $f(g(x)) - g(f(x)) = 0$

14. Solve the trig equation $7\sin 2x + 5\cos x = 0$ for $0 \le x \le 360°$

15. A loch contains 16.7 tons of pollution. Each week treatment by water Authorities removes 64% of the pollution present. A local factory is allowed to dump 3.5 tons of waste in the loch at the end of each week.
Find (a) the amount of waste in the loch at the end of weeks one to six.
(b) the long term state of the loch.

16. Find $\frac{dy}{dx}$ where a) $y = \sqrt{x}(x+3)$ b) $y = \frac{2x+9}{\sqrt{x}}$

17. Find the equation of the tangent to the curve $y = (1-2x)^2$ at x = 1

18. If $f(x) = 3x + 2, g(x) = 1 - 2x$ and $h(x) = 4x + 1$ find

(a) $f(g(x))$ b) $g(h(x))$ c) $h(f(x))$

19. Find the stationary points of the function $f(x) = x^4 - 8x^2 + 16$ and determine the nature of each.

20. If $\sin A = \frac{4}{5}$ and $\tan B = \frac{5}{12}$ where A and B are both acute find the values of

(i) $\sin(A+B)$ (ii) $\cos(A-B)$

21. The lines $px + 4y - 1 = 0$ and $y = 2x$ are perpendicular to each other. Find the value of p.

22. State the largest feasible domain for each of these functions

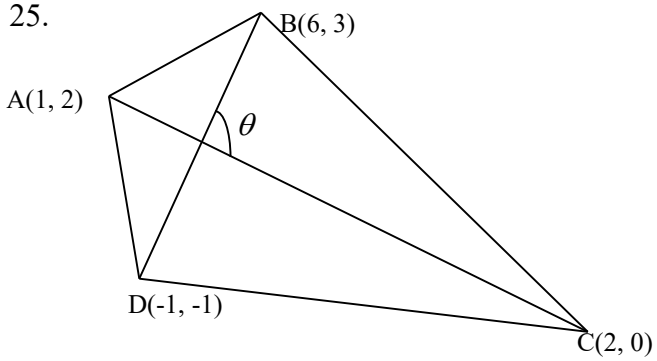
(a) $f(x) = \frac{1}{x-1}$ (b) $f(x) = \sqrt{(2x-6)}$ (c) $f(x) = \frac{3}{x^2 - 2x - 8}$

(d) $f(x) = \sin^{-1} x$

23. Show that the triangle ABC is an isosceles triangle when A (-1, 4) B (1, 7) and C (-2, 9).

24. A yellow circle Y has centre A (1, 3) and radius $\sqrt{5}$. A red circle R has centre B(11, 8) and radius $4\sqrt{5}$.
    (a) Show that Y just touches R
    (b) Find the co-ordinates of the point W where Y touches R.
    (c) Find the equation of the common tangent to both circles, which passes through W.

25.



Find the size of the angle between diagonals AC and DB marked θ
(N.B. this is not necessarily a kite)

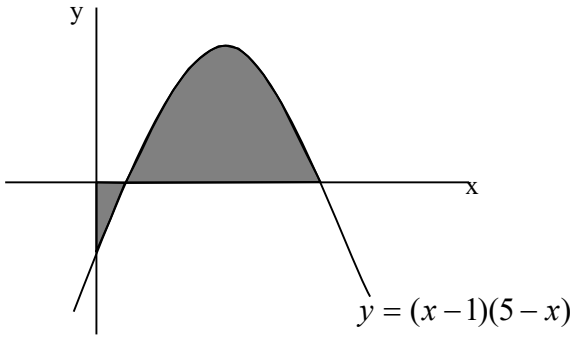
26. A young ex-fisherman, deciding to open his own trout farm, puts 1000 fresh trout into his new pond. He reckons that he will lose 30% of the fish each month due to fish being caught and dying of natural causes.
    He can now only afford to breed 100 fish to add to the pond at the end of each month, and he wonders if his new venture will be worth carrying out.
    The trout society inform him that if ever the number of fish falls below 350 in a trout farm like his, then he should give up. Will he eventually have to give up or will he remain in operation?

27. If A(-4, 0), B(0, -2) and C(6, 6) find the equations of
(a) the median drawn from C to AB
(b) the altitude from B to AC
(c) the perpendicular bisector of BC

28. Prove that the gradient of the tangent to the curve $y = x^3 - 6x^2 + 12x + 1$ is never negative. Find the point on the curve where the gradient is zero.

29. Integrate (a) $x^4 + x^2 - x + 5$ (b) $(x+3)(x-1)(x+1)$ (c) $\dfrac{2x^5 - 3x}{x^{-8}}$

30. Calculate the shaded area in this diagram



31 (a) Find the point of intersection of the tangents to parabola $y = x^2 + 2$ at points P(-2, 6) and Q(1, 3).
    (b) Hence calculate the area bounded by the tangents and the arc PQ.

32. Calculate the area bounded by these two curves $y = 10 - x^2$ and $y = (x-2)^2$ by finding their points of intersection first.

1. a) $-\frac{3}{2x^2}-\frac{4}{x^3}$ b) $2x-\frac{2}{x^3}$ 2. $2y=-x+2$ 3. a) $\frac{120}{169}$ b) $-\frac{119}{169}$

4. $f'(x)=(x-2)^2$ and $f'(x)\geq 0$ for all x 5. $c=1$ or $c=-1$

6. $x=60, 126.9, 300, 233.1$ 7. Proof 8. see teacher

9. $x=6cm$ 10. $k=1$ or $k=-2$ 11. $y=2x-1$ $(1,1)$ and $\left(-\frac{1}{5},-\frac{7}{5}\right)$

12. a) $y-1=-\frac{3}{4}(x-5)$ b) proof leading to $x^2+4x+4=0\Rightarrow(x+2)^2=0\Rightarrow x=-2$

13. a) $f(g(x))=x^2-x+1$ $g(f(x))=x^2+7x+9$ b) $x=-1$

14. $x=90, 200.9, 270, 339.1$ 15. a) 9.51, 6.92, 5.99, 5.65, 5.53, 5.49 . . . b) 5.47

16. a) $\frac{3}{2}\sqrt{x}+\frac{3}{2\sqrt{x}}$ b) $\frac{1}{\sqrt{x}}-\frac{9}{2\sqrt{x^3}}$ 17. $y=4x-3$

18. a) $5-6x$ b) $-1-8x$ c) $12x+9$

19. min tp at (-2, 0) max tp at (0, 16) min tp at (2, 0) 20. (i) $\frac{63}{65}$ (ii) $\frac{56}{65}$

21. $p=2$ 22. a) $x\neq 1$ b) $x\geq 3$ c) $x\neq -2$ or $x\neq 4$ d) $-1\leq x\leq 1$

23. $AB=BC=\sqrt{13}$ $\therefore$ triangle is isosceles 24. a) $r_Y+r_R=AB=5\sqrt{5}$ b) W(3, 4) c) $y=-2x+10$

25. $\vartheta=93.1$ 26. $L=333$ and since $333<350$ he will fail

27. a) $8y=7x+6$ b) $3y=-5x-6$ c) 4y=-3x+17

28. $\frac{dy}{dx}=3(x-2)^2\geq 0$ for all x (2, 9)

29. a) $\frac{1}{5}x^5+\frac{1}{3}x^3-\frac{1}{2}x^2+5x+C$ b) $\frac{1}{4}x^4+x^3-\frac{1}{2}x^2-3x+C$ c) $\frac{1}{7}x^{14}-\frac{3}{10}x^{10}+C$

30. 13 square units 31. a) $y=-4x-2$ and $y=2x+1$ giving point $\left(-\frac{1}{2},0\right)$ b) $2\frac{1}{4}$ sq units

32. $21\frac{1}{3}$ square units